UNIV 2014 Assignment #3

Deadline: April 10, 2019

1) [Vector spaces] Prove or disprove the following statements.
   a) $\mathbb{R}^2$ with the usual operations of scalar multiplication and vector addition is a vector space when the field F is $\mathbb{R}$.
   b) $\mathbb{R}^2$ with the usual operation of scalar multiplication but with vector addition given by x "+" y = 2(x + y) is a vector space when the field F is $\mathbb{R}$.
2) [Subspaces] Determine whether the following subsets are subspaces of the given vector spaces.
   a) U=$\{A \in R^{2X2} \mid A^T = A\}$, $\mathbb{R}^{2X2}$ ($\mathbb{R}^{2X2}$ is the vector space of all real 2 by 2 matrices). [Also find the dimension of $\mathbb{R}^{2X2}$.]
   b) W=$\{(x, y, z) \in R^3 \mid x \geq y \geq z\}$, $\mathbb{R}^3$
3) [Linear independence] Let P_1 and P_2 be non-zero column vectors and A P_1=2 P_1 and A P_2=3 P_2. Show that P_1 and P_2 are linearly independent.
4) [Basis and dimension] Given 4 vectors a, b, c,d.
   a=$[1 \quad 1 \quad -2 \quad 0 \quad -1]$, b=$[1 \quad 2 \quad 0 \quad -4 \quad 1]$,
   c=$[0 \quad 1 \quad 3 \quad -3 \quad 2]$, d=$[2 \quad 3 \quad 0 \quad -2 \quad 0]$
   a) Find the matrix A consisting of rows that are the vectors a, b, c, d and find the echolon form E of the matrix A.
   b) Find a basis for the vector space spanned by the vectors a, b, c, d. The basis is given by the rows of the matrix E.
   c) Find the dimension of that vector space.
   d) Find the matrix A^T (its columns are the vectors a, b, c, d) and find the echolon form F of the matrix A^T.
   e) Find another basis for the vector space spanned by the vectors a, b, c, d. The basis is given by the pivotal columns of A^T.
5) [Fundamental subspaces] Let A=$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & -2 \end{bmatrix}$.
   a) Find the echolon form of A.
   b) Find a basis for the row space.
   c) Find a basis for the column space.
   d) Find a basis for the nullspace.
   e) State the dimensions of the row space, column space, and the nullspace.
6) [Fundamental Theorem of Linear Algebra]

   Let A=$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & -1 \\ 9 & 5 & 1 \\ 9 & 8 & 7 \end{bmatrix}$.

   a) Find a basis for the the row space and a basis for the nullspace.
   b) What is the sum of dimensions of row space and the nullspace equal to?
   c) Verify that all vectors in the basis of the nullspace are orthogonal to all vectors in basis of the row space.